Group Synchrony in an Experimental System of Delay-coupled Optoelectronic Oscillators

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Abstract—The study of group synchronization of delay-coupled dynamical systems is of interest in the context of physical and biological systems. The delay-coupled nodes or oscillators are placed into groups based on different parameters or governing equations. In this case, it has been shown theoretically that nodes in the same group may identically and isochronally synchronize with the other nodes in the group, even if there is no direct intra-group coupling [1, 2]. We report experimental observations of *group synchrony* in a network of four nonlinear optoelectronic feedback loops that are segregated into two groups of two nodes each. Both nodes in a single group have identical parameters, which may be different from the parameters in the other group. All of the nodes are coupled to each node in the other group, but there is no intra-group coupling. We find that each node will identically synchronize with the other node in its group, but will have distinctly different dynamics than the nodes in the other group, to which it is directly coupled. We compare the experimental results with numerical simulations.

1. Introduction

The synchronization of dynamical systems is an interesting subject for understanding many natural systems such as phenomena in ecology, physiology, epidemiology, and collective behavior of organisms. Additionally, achieving or avoiding synchrony is a key feature in many engineering applications such as sensors, transportation systems, and structure design. Optoelectronic feedback loops have been used to generate a variety of dynamical behaviors in experiments, including chaotic and pulsed dynamics [3, 4]. These experimental systems can be used to understand synchronization properties between coupled oscillators [5].

When dynamical systems are coupled and placed into two groups such that each node has identical equations and parameters as the other nodes in its group, there are three types of synchronous behavior possible that are described in this paper: *identical synchrony, cluster synchrony,* and *group synchrony* [2]. If the equations and parameters of the two groups are the same, it is possible for the groups to display *cluster synchrony*, where all the nodes in a given group are isochronally synchronized to the other nodes in the group, but the two groups are not necessarily isochronally synchronized. If all nodes are isochronally synchronized, this *identical synchrony* is a special case of cluster synchrony. If the equations or parameters of one group differ from those of the other group, *group synchrony* is possible, in which case, like cluster synchrony, the nodes will isochronally synchronize within a group, but not between the groups. Group synchrony is a generalization of cluster synchrony in which the nodes in one group need not be identical to those in the other group.

2. Experimental Setup

We construct a system of four nodes, where each node is an oscillator constructed from an optoelectronic feedback loop. The nodes are separated into two groups, A and B, with two nodes in each group, as shown in Fig. 1a. Both nodes in group A have identical parameters, and both nodes in group B have identical parameters, but the parameters of group A may or may not be identical to the parameters of group B. The primary distinguishing feature between groups A and B is that nodes in group A are connected to each node in group B and vice versa, but the nodes are not directly connected to the other node in the same group. Each loop is a nonlinear oscillator and has independent parameters that can be varied to create a wide range of dynamical behaviors, from fixed point to periodic to quasi-periodic to chaotic. The oscillators are coupled with directional links, enabling unidirectional or bidirectional cou-

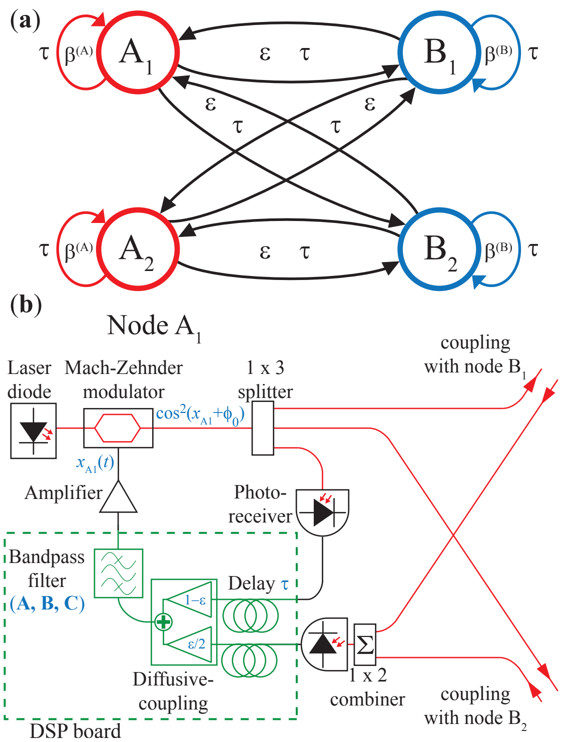

Figure 1: a) Schematic of four nodes separated into two groups, A (red) and B (blue). Except for the coupling strength, β, all parameters are identical for all nodes. The β values for each group may be different from the other group, but the value of β is identical for each node in the same group. b) Experimental set up of a single node, showing coupling to the other nodes. Optical connections are shown in red, electronic in black. The values for β are controlled by a scale factor on the digital signal processing (DSP) board.

pling between every pair of nodes. For the investigations presented here, the coupling was adjusted so that there was bidirectional coupling between each pair of coupled nodes.

Each feedback loop consists of an optical part and an electronic part, as shown by the red and black lines in Fig. 1b. The nonlinearity is created by a Mach-Zehnder modulator (MZM), whose optical output is a cosine-squared function of a voltage input. The coupling is implemented optically, and is adjusted so that each connection has the same coupling strength. In this particular experimental realization of the optoelectronic feedback loop, each node has a digital signal processing (DSP) board, usually used for audio signal processing, that implements the feedback and coupling delays, coupling strength, filter, and an overall gain factor to control the feedback strength, β. The parameter β is the one that is varied in this investigation, and although β is an overall feedback strength that is composed of many factors including the gain of the photoreceivers and the amplifier after the DSP board, we adjust the feedback strength with an amplification factor on the DSP board only. Here, the feedback and coupling delays are equal to the same delay, τ, which is set on the DSP board to 1.4 ms. The filter is a digital two-pole bandpass filter from 0.1 to 2.5 kHz with a sampling rate of 24 kS/s. The coupling is diffusive coupling with a global coupling strength of $\varepsilon = 0.8$, where the coupling strength of any given link is $\varepsilon/2 = 0.4$, since there are two incoming connections to each node. This means that for this coupling configuration with two incoming signals to each nodes, the feedback signal is scaled by an additional factor of $1 - \varepsilon = 0.2$.

The feedback strength, β, can be varied from 0 to 10, but is maintained constant for a given realization. This is the parameter that is adjusted in these experiments and simulations described here. All other parameters are held constant for all realizations. For each measurement taken, the system is started with no feedback or coupling, from random initial conditions. Then feedback is enabled, with no coupling. Finally, coupling is enabled, and there is a transient to synchrony, if synchrony can be observed.

3. Mathematical Model

The experimental setup of a single feedback loop can be mathematically represented by the block diagram in Fig. 2. It can be described by time delay differential equations [6]:

$$\dot{u}_i(t) = \mathbf{A}u_i(t) - \mathbf{B}\beta\cos^2(x_i(t-\tau) + \phi_0), \tag{1}$$

$$x_i(t) = \mathbf{C}[(u_i(t) + \varepsilon\sum_j K_{ij}(u_j(t) - u_i(t))], \tag{2}$$

$i = 1, ..., 4$.

A, **B,** and **C** are constant matrices that describe the filter. K is the coupling matrix, given by

$$K = 1/2\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}. \tag{3}$$

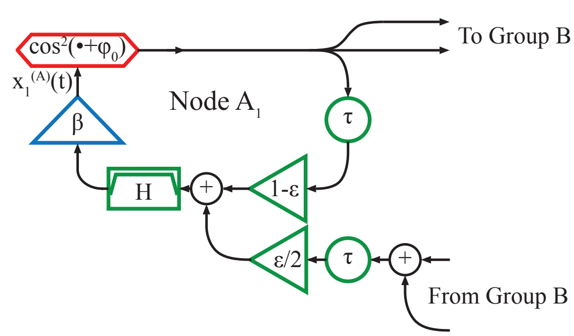

Figure 2: Mathematical block diagram of a single feedback loop. Each loop consists of a nonlinearity, diffusive coupling with coupling strength ε, a time delay τ that is the same for both feedback and coupling signals, a bandpass filter represented by H, and an overall feedback strength β.

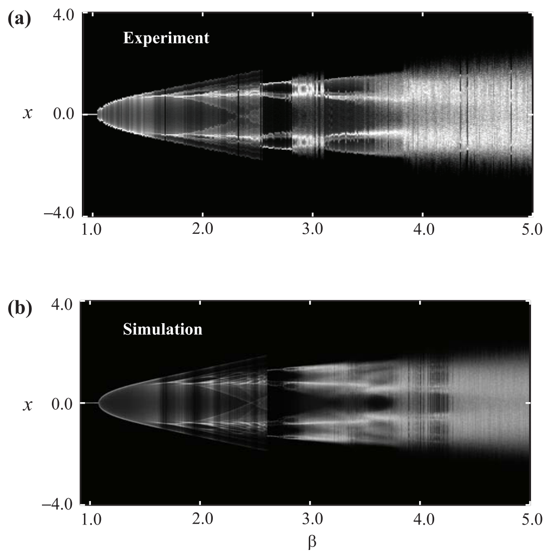

Figure 3: Bifurcation diagrams in (a) experiment and (b) simulation for a single, uncoupled node [8]. The range of β is not as wide as that reported in this paper, and the filter is 0.1-10 kHz, rather than 0.1-2.5 kHz.

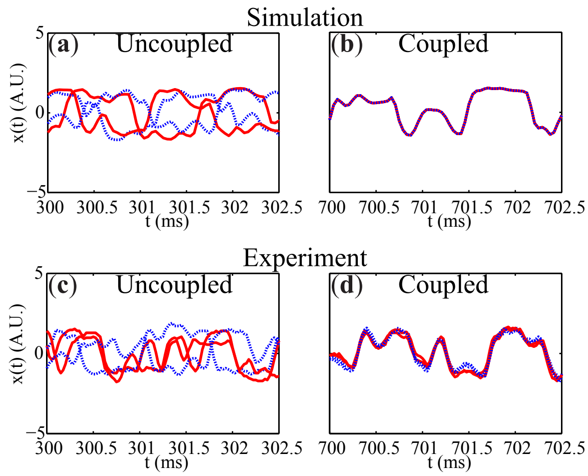

Figure 4: Simulation (a,b) and experimental (c,d) time traces. For all values of $\beta = 3.3$, the four nodes display *identical synchrony*. The dynamics of the coupled nodes (a,c) are qualitatively the same as the dynamics of the uncoupled nodes (b,d).

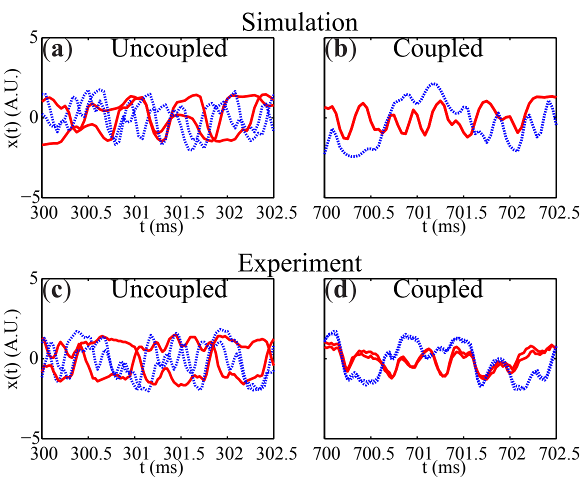

Figure 5: Simulation (a,b) and experimental (c,d) time traces. For $\beta_A = 3.3$ and $\beta_B = 4.7$, the dynamics of the four nodes split into two distinct groups, A (red, solid) and B (blue, dashed), and we observe *group synchrony*. The coupled dynamics display a slightly larger amplitude for Group B than for Group A.

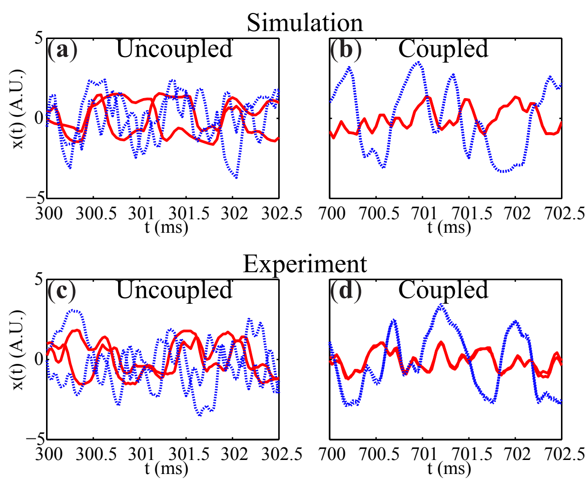

Figure 6: Simulation (a,b) and experimental (c,d) time traces. For $\beta_A = 3.3$ and $\beta_B = 7.6$, the dynamics of the four nodes split into two distinct groups, A (red, solid) and B (blue, dashed), and we observe *group synchrony*. Here, the dynamics of Group A are clearly different qualitatively from the dynamics of Group B.

Numerical simulations use a discrete time implementation of these differential equations, as described in [7]. The simulations produce remarkable agreement to the experimental results for the variety of dynamical behaviors that can be observed. Bifurcation diagrams for a single, uncoupled node of this system with a filter from 0.1 to 10 kHz have been reported in [8] and are shown in Fig. 3. It can be seen that these systems can display a wide variety of dynamical behaviors (fixed point, periodic, quasiperiodic, and chaotic), and that the dynamics in simulation correspond well to the dynamics observed in the experiments.

4. Experimental Results

When we couple the four nodes as shown in Fig. 1, we observe *identical synchrony, group synchrony,* or *cluster synchrony*, as described above. For $\beta_A = \beta_B = \beta$ for small values of β, the four nodes display identical syn-

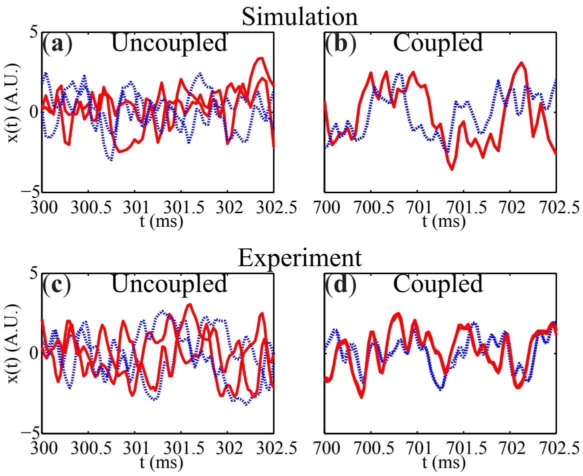

Figure 7: Simulation (a,b) and experimental (c,d) time traces. For $\beta_A = 7.6$ and $\beta_B = 6.6$, the dynamics of the four nodes split into two distinct groups, A (red, solid) and B (blue, dashed), and we observe *group synchrony*. The dynamics of Group A have only a slightly larger amplitude than the dynamics of Group B.

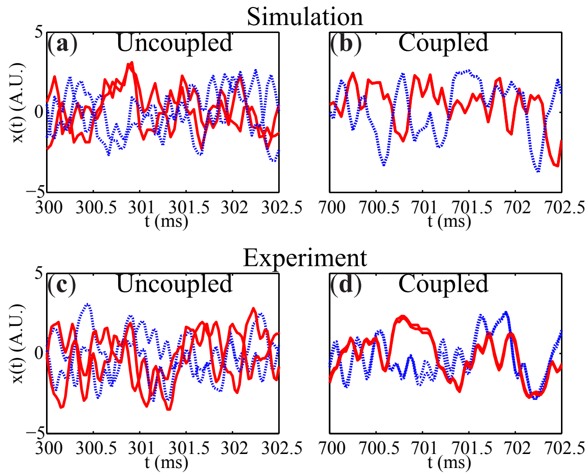

Figure 8: Experimental (a,b) and simulation (c,d) time traces. For all values of $\beta = 7.6$, the four nodes display *cluster synchrony*, but not identical synchrony, despite the fact that all four nodes have identical parameters. The dynamics of the coupled nodes (a,c) are qualitatively the same as the dynamics of the uncoupled nodes (b,d).

chrony (Fig. 4). As β_A and β_B are made non-identical, we observe group synchrony (Figs. 5-7). For larger values of identical βs, however, we see cluster synchrony and not identical synchrony (Fig. 8). Theoretical analysis indicates that for small values of identical βs (Fig. 4), we expect bistability between cluster and identical synchrony. However, this is not a trivial observation to make experimentally. For $\beta_A \neq \beta_B$, we observe group synchrony, in general.

5. Conclusions

We have constructed an experiment with four optoelectronic nonlinear oscillators that are coupled together in a two-group configuration. In this configuration, we observe group synchrony, in which the dynamics of the nodes within a particular group are identically synchronized. For the case where the parameters of the two groups are different, we observe *group synchrony*, where the dynamics of the two groups are qualitatively and quantitatively different. For the cases when the parameters of all nodes are identical, we observe *identical* (small β) or *cluster* (large β) *synchrony*. Numerical simulations show similar behaviors for the dynamics of the nodes, both for the case where the nodes are uncoupled and for the case where the nodes are coupled into the two-group configuration.

Acknowledgements

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