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Network structural properties, communication models and traffic dynamics

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Abstract— **In this paper, we expound some of our recent results concerning the characterization of the relationship between the network topology and traffic dynamics taking place over it [1], [2]. We use a model of network generation that allows the transition from random to scale free networks. Specifically, we consider three different topological types of network: random, scale-free with $\gamma = 3$, scale-free with $\gamma = 2$. We firstly compare the performance of these networks in terms of *throughput* and average *delivery time*, under the hypothesis of constant transmission rates and infinite queue lengths at the network vertices. Interestingly, scale-free networks that are characterized by shorter characteristic-path-length (the lower the exponent, the lower the path-length), show worst performances in terms of communication. We present an explanation of this in terms of changes in the *load* distribution, defined as the number of shortest paths going through a given vertex. Then the issue is addressed of how the traffic behavior on the network is influenced by the variable factors of the transmission rates and queue length restrictions at the network vertices. We show that these factors can induce drastic changes in the *throughput* and *delivery time* of the network and are able to counter-balance some undesirable effects due to the topology.**

I. Introduction

Much research effort has been spent recently in understanding the relationship between network topological features and communication performances. As a first approximation, it would be natural to make the most general hypothesis about the structure of the underlying network, that is, to think of it as a random graph. Unfortunately, real networks show statistical properties that are far from being completely random. The most important difference is that they have typically power law degree distributions with exponents between 2 and 3 [3]. Thus, in what follows, we consider three different topologies, in the order: random, scale-free with $\gamma = 3$, scale-free with $\gamma = 2$.

By making use of a packet transport model that has been widely studied in the literature (see [4], [5], [6] for further details), we compare the main indicators of the network performance, specifically the delivery time and the number of delivered packets (or throughput), as the underlying topology is varied.

II. Network generation model

In order to cause the transition from random to scale-free network we use the static model recently introduced in [7]. Vertices are indexed by an integer i, for $(i = 1...., N)$, and assigned a *weight* or *fitness* $p_i = i^{-\alpha}$ where α is a parameter between 0 and 1. Two different vertices are selected with probabilities equal to the normalized weights, $p_i / \sum_k p_k$ and $p_j / \sum_k p_k$ respectively and an edge is added between them unless one exists already. This process is repeated until M edges are made in the system leading to the mean degree $\langle k \rangle = 2M/N$. This results in the expected degree at vertex i scaling as $k_i \sim (\frac{N}{i})^\alpha$ [7]. We then have the degree distribution, i.e. the probability of a vertex being of degree k, given by $P(k) \sim k^{-\gamma}$ with $\gamma = 1 + \frac{1}{\alpha}$. Thus, by varying α, we can obtain the exponent γ in the range, $2 < \gamma < \infty$. Moreover the ER graph is generated by taking $\alpha = 0$.

It is worth noting that the static model described here, can be considered as an extension of the standard ER model for generating *random-scale free networks*, i.e. networks with prescribed degree distribution, but completely random with respect to all the other features.

III. Load distribution in Networks

One of the main parameters of vertex centrality is *betweenness centrality* defined as the number of shortest paths between pairs of nodes crossing a given vertex [8]. Taking this index as a starting point, Goh *et al.* [7] [9], defined the *load* at each vertex v, say $l(v)$, as the number of packets passing through it, under the assumption that every node sends a packet to every other node in the network and that packets move in parallel from origin to destination through the geodesic, i.e. the shortest path between them. This implies that for each shortest path between a given couple of vertices, there is a packet passing along it; in the case that packets encounter a branching point at which there is more than one shortest path toward the destination, they would be divided by the number of branches at the branching point.

As pointed out in [1], in comparison to random graphs, scale free networks are characterized by:

- Lower average load (averaged over all the network vertices).
- Higher load standard deviation.

Intuitively, the presence of hubs in scale free networks, results in a shorter average distance between vertices. On the other hand, the increase in the load standard deviation indicates that this happens at the expense of the fairness of the network resources exploitation, with a relatively few vertices drawing most of the network traffic. As we will show next, such a phenomenon results to be particularly noxious to the communication dynamics taking place over the network.

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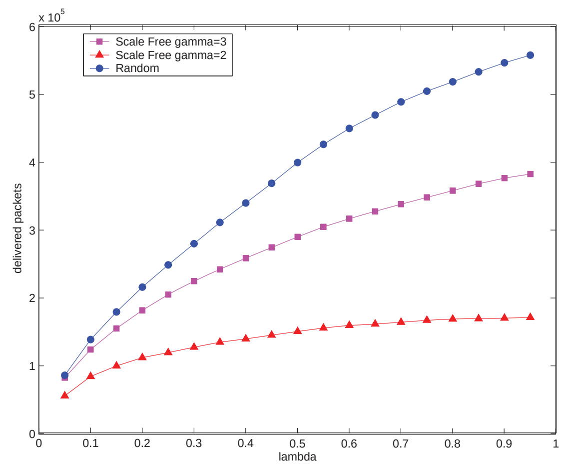

Fig. 1. Number of delivered packets versus the generation rate, λ. Three different networks, random, scale-free with $\gamma = 3$, scale-free with $\gamma = 2$, have been compared, while keeping a fixed number of vertices (500) and the edge degree (3 per node).

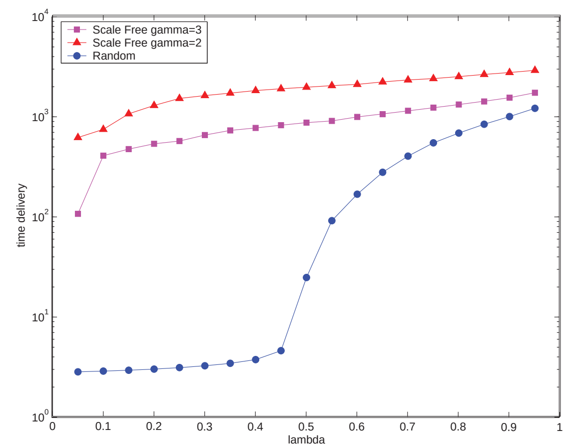

Fig. 2. Delivery times versus the generation rate, λ. Three different networks, random, scale-free with $\gamma = 3$, scale-free with $\gamma = 2$, have been compared, while keeping fixed the number of vertices (500) and the edges (3 per node)

IV. Model of Network Data Traffic

We use the family of *Erramilli* interval maps as the generator for each LRD traffic source, (Erramilli *et al.*, 1994),[10] within the network, as further explained in [1].

We assume, the network involves two types of nodes: hosts and routers. The first are nodes that can generate and receive messages and the second can only store and forward messages. The density of hosts $\rho \in [0, 1]$ is the ratio between the number of hosts and the total number of nodes in the network (in this paper we take $\rho = 0.16$). Hosts are randomly distributed throughout the network.

A routing algorithm is needed to model the dynamic aspects of the network. Packets are created at hosts and sent through the lattice one step at a time until they reach their destination host.

The routing algorithm operates as follows: (1) First a host creates a packet following a distribution defined by the chaotic map (LRD) described above. If a packet is generated it is put at the end of the queue for that host. This is repeated for each host in the lattice. (2) Packets at the head of each queue are picked up and sent to a neighboring node selected according to the following rules: (a) A neighbor closest to the destination node is selected. (b) If more than one neighbor is at the minimum distance from the destination, the link through which the smallest number of packets have been forwarded is selected. (c) If more than one of these links shares the same minimum number of packets forwarded, then a random selection is made.

This process is repeated for each node in the lattice. The whole procedure of packet generation and movement represents one time step of the simulation.

V. Effects on network performance of varying the underlying topology

We have compared three different topologies: random, scale-free with $\gamma = 3$, scale-free with $\gamma = 2$ in order to evaluate the effects of the underlying topology on the network performances.

The networks we consider have different degree distributions but are characterized by the same number of available *resources*, that is by the same number of vertices and edges. Where the resulting network is not fully connected, we have only considered the giant component.

In Fig.1 the number of delivered packets, or throughput, has been plotted as a function of the generation rate, λ, for the three considered topologies. Scale-free networks show the least effective performances in that the number of delivered packets is lower than for random networks, with Poisson degree distribution. For scale-free networks, those with $\gamma = 2$ are still less effective than those with $\gamma = 3$. Though our analysis is purely qualitative, we would like to point out that the real Internet has a power-law degree distribution with $\gamma = 2.2$ [11].

Notice that the differences among the different considered topologies, increase for higher values of λ. In particular random networks seem to behave better than other networks under high traffic rates. It is worth noting that this is in strong agreement with results shown in [12].

In Fig.2, the delivery time for packets to reach their destination has been plotted versus the generation rate, λ. The results are in accordance with those for throughput: the highest delivery time have been achieved for random networks, the lowest for scale-free networks with $\gamma = 2$.

The reason for this is that packets that are stored in the routers' queues without being delivered to their destination, increase the time needed for other packets to reach their destination. Moreover scale-free networks show a vanishing value of the critical load λ, i.e. the value of λ at which a phase-transition occurs [4], with respect to random graphs. Consequently, although scale-free networks are characterized by a shorter characteristic-path-length [13], they show worst performances in terms of communication.

It is somewhat surprising that the structure of scale-free graphs, which are ubiquitous in nature, does not lead to any benefit but rather a worsening in terms of the end-to-end performance. In particular, the characteristic parameters known as *throughput* and *delivery time* are considerably affected by the congestion at the network hubs. This is counter-intuitive when one considers that the shortening of the distances in the network might result in a reduction of the delivery time and thus an increase of the throughput.

This interesting phenomenon is analogous to the *paradox of heterogeneity* [14], which has been observed in the context of synchronizability of scale-free networks.

VI. Topology-aware communication models

Sofar we have considered no differences in the communication behavior among nodes characterized by different degree. Yet, it is unrealistic to assume that resources such as bandwidth are uniformly distributed among the network nodes in strongly heterogenous networks. Instead, it is very likely that hubs, which are characterized by a high number of incoming and outgoing links, are found to play a fundamental role in communication over the network. They are typically characterized by having higher server strength transmission rates and larger buffers than more peripheral nodes.

For these reasons, in [**?**] we have introduced the following *topology-aware* communication model: (i) the transmission rate r is assumed to scale with the degree at each vertex i, $k(i)$, as: $r(i) = c_1 k(i)^\alpha$ (note that in the particular case where $\alpha = 0$, we recover the original case, with all the nodes having the same transmission rates); (ii) the maximum queue length (i.e., the buffer size) is no longer assumed infinite but is taken to scale with the degree at each vertex i, $k(i)$, as: $q(i) = c_2 k(i)^\beta$.

In what follows, we analyze separately, by means of numerical simulations, the effects of varying α and β on the network communication performance. As a representative case, we assume $c_1 = 1$ and $c_2 = 50$. Similar behavior was observed for other values of c_1 and c_2.

The routing algorithm is the same as the one described above, with the difference that at each iteration of the algorithm, a third step is considered. Namely, packets at the head of each queue, exceeding its maximum capability, are dropped.

VII. Network Performance

Using the network model and traffic generator detailed above, simulations were carried out to analyse various aspects of end-to-end performance for two types of network. Namely, results for random graphs have been paired with those of scale-free graphs with $\gamma = 3$. We have calculated the corresponding output for scale-free graphs with $\gamma = 2$ and have found that the differences in behaviour with the alternative value $\gamma = 3$ are negligible by comparison with the behaviour of the random graph, and so the third set of comparisons is not repeated here.

In Fig. 3 we see that random graphs respond more quickly with smaller delivery times as α increases (from zero). Similarly, it is observed the communication is much more efficient in terms of delivered packets at high loads (or generation rate) as α increases. The number of delivered packets, instead, is observed to be unaffected by the buffer sizes at the nodes, being mainly determined by the network topology. Finally, in Fig. 4, the number of dropped packets is observed to decrease as the buffer sizes are scaled more sensitively with vertex degree. (More evidence can be found in [1],[**?**] where further simulation results are reported.)

VIII. Conclusions

We have shown how topological transitions in a given network from random to scale free affect the load distribution on the network itself. In particular, we characterised such load distribution in terms of the average load and its standard deviation. We observed that as the topological transition takes place, the network performance worsens and the load tends to become more localised (higher standard deviation). Moreover, by introducing a topology-aware communication model, we showed how it is possible to counter-balance some undesirable effects due to the topology.

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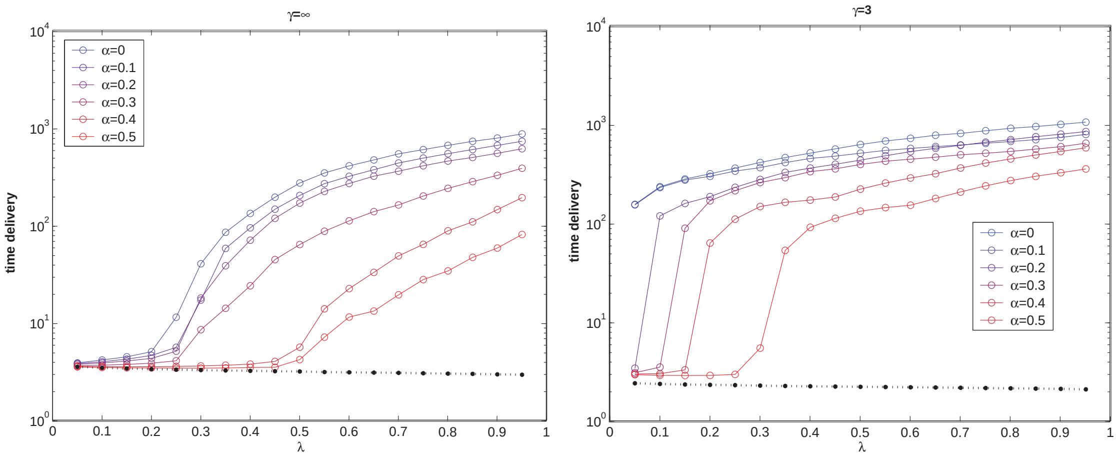

Fig. 3. Delivery time versus the generation rate, λ. The network is a (a) random graph ($\gamma = \infty$), (b) scale-free graph ($\gamma = 3$) with number of nodes $N = 512$ and number of edges $M = 2N$. We show the effects of varying the transmission rates $r(i)$ at node i, according to the law $r(i) = k(i)^{\alpha}$, for α ranging between 0 and 0.5 (blue to red). The black dotted line represents the free regime at $\alpha = 1$.

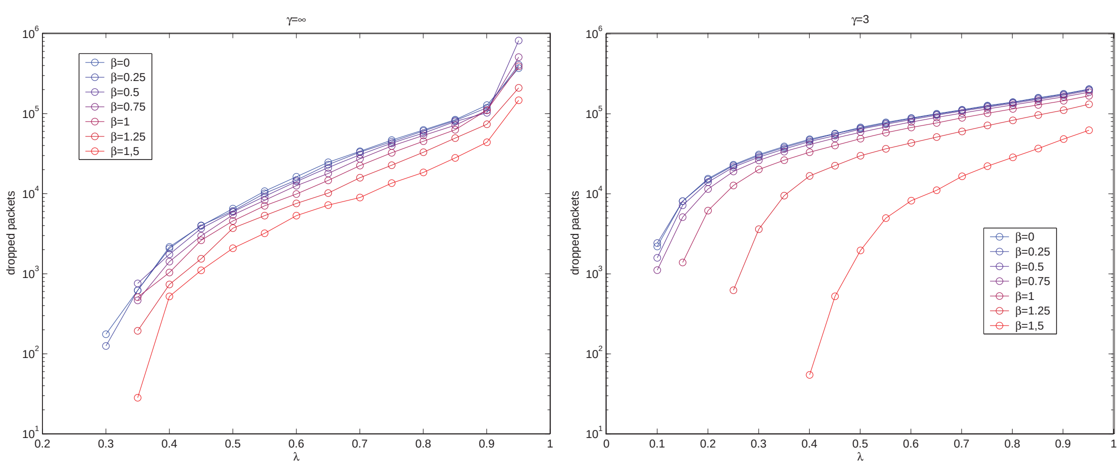

Fig. 4. Number of dropped packets versus the generation rate, λ. The considered network is scale-free with $\gamma = 3$ and $\gamma = \infty$, the number of nodes being $N = 512$ and number of edges $M = 2N$. We show the effects of varying the queue length $q(i)$ at node i, according to the law $q(i) = 50k(i)^{\beta}$, for β ranging between 0 and 1.5.